

Name: _____

Instructor: _____

Math 10560, Practice Exam 2.
October 12, 2026

- The Honor Code is in effect for this examination. All work is to be your own.
- No calculators.
- The exam lasts for 1 hour and 15 min.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 17 pages of the test.
- Each multiple choice question is worth 7 points. Your score will be the sum of the best 10 scores on the multiple choice questions plus your score on questions 13-16.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!					
1.	(a)	(b)	(c)	(d)	(e)
2.	(a)	(b)	(c)	(d)	(e)
.....					
3.	(a)	(b)	(c)	(d)	(e)
4.	(a)	(b)	(c)	(d)	(e)
.....					
5.	(a)	(b)	(c)	(d)	(e)
6.	(a)	(b)	(c)	(d)	(e)
.....					
7.	(a)	(b)	(c)	(d)	(e)
8.	(a)	(b)	(c)	(d)	(e)
.....					
9.	(a)	(b)	(c)	(d)	(e)
10.	(a)	(b)	(c)	(d)	(e)
.....					
11.	(a)	(b)	(c)	(d)	(e)
12.	(a)	(b)	(c)	(d)	(e)

Please do NOT write in this box.

Multiple Choice _____

13. _____

14. _____

15. _____

Total _____

Name: _____

Instructor: _____

Multiple Choice

1.(7 pts.) Use a rationalizing substitution to evaluate the integral

$$\int \frac{\sqrt{x+1}}{x+2} dx.$$

Let

$$u = \sqrt{x+1}.$$

Then

$$u^2 = x+1, \quad x = u^2 - 1, \quad dx = 2u \, du.$$

Also,

$$x+2 = u^2 + 1.$$

Therefore,

$$\int \frac{\sqrt{x+1}}{x+2} dx = \int \frac{u}{u^2+1} (2u) \, du = \int \frac{2u^2}{u^2+1} \, du.$$

Since

$$\frac{u^2}{u^2+1} = 1 - \frac{1}{u^2+1},$$

we obtain

$$2 \int \left(1 - \frac{1}{u^2+1} \right) du = 2u - 2 \arctan u + C.$$

Substituting back $u = \sqrt{x+1}$, we get

$$2\sqrt{x+1} - 2 \arctan(\sqrt{x+1}) + C.$$

- (a) $2\sqrt{x+1} - 2 \ln(x+2) + C$
- (b) $2\sqrt{x+1} - \arctan(\sqrt{x+1}) + C$
- (c) $2\sqrt{x+1} - 2 \arctan(\sqrt{x+1}) + C$
- (d) $2\sqrt{x+1} + 2 \arctan(\sqrt{x+1}) + C$
- (e) $\sqrt{x+1} - 2 \ln(\sqrt{x+1}) + C$

Name: _____

Instructor: _____

2.(7 pts.) If you expand $\frac{2x+1}{x^3+x}$ as a partial fraction, which expression below would you get?

- $\frac{2x+1}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$
- Multiplying the above equation by $x(x^2+1)$, we get $2x+1 = A(x^2+1) + x(Bx+C) = Ax^2 + A + Bx^2 + Cx = (A+B)x^2 + Cx + A$.
- Comparing coefficients, we get $A = 1$, $C = 2$, and $A + B = 0$. Therefore $B = -A = -1$.
- The partial fractions decomposition of $\frac{2x+1}{x(x^2+1)}$ is therefore $\frac{1}{x} + \frac{-x+2}{x^2+1}$.

(a) $\frac{2}{x} + \frac{1}{x^2+1}$

(b) $\frac{-1}{x^2} + \frac{1}{x+1}$

(c) $\frac{-1}{x} + \frac{x}{x^2+1}$

(d) $\frac{1}{x} + \frac{-x+2}{x^2+1}$

(e) $\frac{-2}{x} + \frac{1}{x^2+1}$

Name: _____

Instructor: _____

3.(7 pts.) Use Simpson's rule with $n = 4$ to approximate the integral $\int_0^4 f(x)dx$ where a table of values for the function $f(x)$ is given below.

x	0	1	2	3	4
$f(x)$	2	1	2	3	5

$$\int_0^4 f(x) dx \approx \frac{\Delta x}{3} [f(0) + 4 \cdot f(1) + 2 \cdot f(2) + 4 \cdot f(3) + f(4)] = \frac{1}{3} [2 + 4 + 4 + 12 + 5] = \frac{27}{3} = 9.$$

- (a) 8 (b) 11 (c) 9 (d) 10.4 (e) 9.5

Name: _____

Instructor: _____

4.(7 pts.) Evaluate the improper integral

$$\int_4^{\infty} \frac{1}{(x-2)(x-3)} dx.$$

Using a partial fraction expansion $\int \frac{1}{(x-2)(x-3)} dx = \ln \left| \frac{x-3}{x-2} \right| + C$.

$$\text{Therefore } \int_4^{\infty} \frac{1}{(x-2)(x-3)} dx = \lim_{t \rightarrow \infty} \ln \left| \frac{t-3}{t-2} \right| - \ln \left| \frac{1}{2} \right| = 0 + \ln 2.$$

- (a) the integral diverges (b) $\ln \frac{1}{2}$ (c) $3 \ln 2$
(d) $\ln 3$ (e) $\ln 2$

Name: _____

Instructor: _____

5.(7 pts.) Consider the improper integral

$$\int_{-1}^2 \frac{dx}{(x-1)^2}.$$

One of the following statements is true. Which one?

There is a discontinuity at $x = 1$ so we split into two integrals:

$$\int_{-1}^2 \frac{dx}{(x-1)^2} = \int_{-1}^1 \frac{dx}{(x-1)^2} + \int_1^2 \frac{dx}{(x-1)^2}$$

Evaluating the first part using $u = x - 1$, $du = dx$, $u(-1) = -2$ and $u(t) = t - 1$ we get:

$$\begin{aligned} \int_{-1}^2 \frac{dx}{(x-1)^2} &= \lim_{t \rightarrow 1^-} \int_{-1}^t \frac{dx}{(x-1)^2} \\ &= \lim_{t \rightarrow 1^-} \int_{-1}^t \frac{dx}{u^2} \\ &= \lim_{t \rightarrow 1^-} -\frac{1}{u} \Big|_{-2}^{t-1} \\ &= \lim_{t \rightarrow 1^-} -\frac{1}{t-1} - \frac{1}{2} \\ &\rightarrow -\infty \end{aligned}$$

Since part of the integral diverges, the integral is divergent.

- (a) The integral converges to $-\ln(3)$.
- (b) The integral is divergent.
- (c) The integral converges to $\ln(4)$.
- (d) The integral converges to $-\frac{3}{2}$.
- (e) The integral converges to $\frac{1}{2}$.

Name: _____

Instructor: _____

6.(7 pts.) What can be said about the integrals

$$(i) \int_0^1 \frac{e^x}{x^2} dx;$$

$$(ii) \int_1^\infty \frac{\cos^2 x}{x^2} dx?$$

Integral (i) diverges by the Comparison Theorem since the integrand is greater than $\frac{1}{x^2}$.

Integral (ii) converges by the Comparison Theorem since the integrand is less than $\frac{1}{x^2}$.

- (a) neither integral (i) nor (ii) is improper
- (b) (i) diverges and (ii) converges
- (c) both (i) and (ii) converge
- (d) both (i) and (ii) diverge
- (e) (i) converges and (ii) diverges

Name: _____

Instructor: _____

7.(7 pts.) Which of the following is an expression for the arclength of the curve $y = \cos x$ between $x = -\frac{\pi}{2}$ and $x = \frac{\pi}{2}$?

The arclength formula gives the answer as $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{1 + (-\sin x)^2} dx$.

(a) $2 \int_0^{\frac{\pi}{2}} \sqrt{1 + 2 \sin^2 x} dx.$

(b) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{1 + \sin^2 x} dx.$

(c) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{1 - \sin^2 x} dx.$

(d) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{1 - \cos^2 x} dx.$

(e) $\frac{\pi^2}{2}$

Name: _____

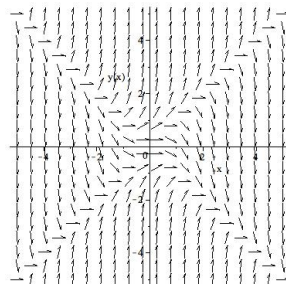
Instructor: _____

8.(7 pts.) Which of the following gives the direction field for the differential equation

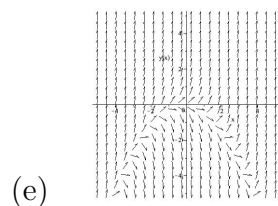
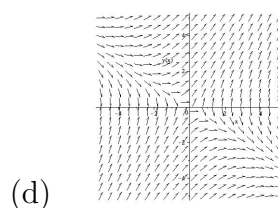
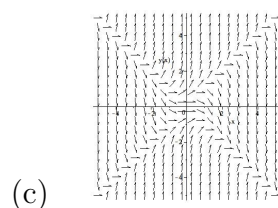
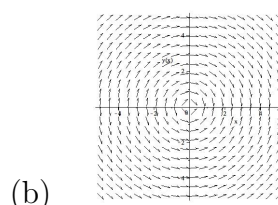
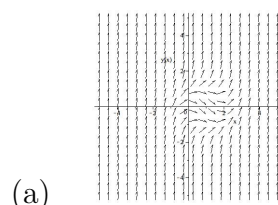
$$y' = y^2 - x^2$$

Note the letter corresponding to each graph is at the lower left of the graph.

For points on the line $y = x$, we must have $y' = 0$. Also for points on the line $y = -x$, we must have $y' = 0$. Hence along both diagonals of the plane, we must have $y' = 0$ and



the answer must be



Name: _____

Instructor: _____

9.(7 pts.) Use Euler's method with step size 0.1 to estimate $y(1.2)$ where $y(x)$ is the solution to the initial value problem

$$y' = xy + 1 \quad y(1) = 0.$$

$$x_0 = 1, \quad y_0 = 0$$

$$x_1 = x_0 + h = 1.1, \quad y_1 = y_0 + h(x_0 y_0 + 1) = 0 + (0.1)(1 \cdot 0 + 1) = 0.1$$

$$\begin{aligned} x_2 = x_1 + h = 1.2, \quad y_2 &= y_1 + h(x_1 y_1 + 1) = 0.1 + (0.1)((1.1)(0.1) + 1) \\ &= 0.1 + 0.1(0.11 + 1) = 0.1 + 0.1(1.11) = 0.1 + 0.111 = 0.211 \end{aligned}$$

(a) $y(1.2) \approx .101$

(b) $y(1.2) \approx .112$

(c) $y(1.2) \approx .211$

(d) $y(1.2) \approx .111$

(e) $y(1.2) \approx .201$

Name: _____

Instructor: _____

10.(7 pts.) Find the solution of the differential equation

$$\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{1+x^2}$$

with initial condition $y(0) = 0$.

Separating variables here gives $\int \frac{dy}{\sqrt{1-y^2}} = \int \frac{dx}{1+x^2}$.

Solving this gives $\arcsin y = \arctan x + C$ and substituting $y(0) = 0$ we find $C = 0$.

Therefore $y = \sin(\arctan x) = \frac{x}{\sqrt{1+x^2}}$.

(a) $y = \frac{x}{1+x}$

(b) $y = \frac{x^2}{\sqrt{1+x^2}}$

(c) $y = \frac{x}{1+x^2}$

(d) $y = \frac{x}{\sqrt{1+x^2}}$

(e) $y = \frac{1}{\sqrt{1+x^2}}$

Name: _____

Instructor: _____

11.(7 pts.) Find the family of orthogonal trajectories to the family of curves given by

$$y = kx^2.$$

$$\frac{dy}{dx} = 2kx$$

For the family of curves given above

$$y = kx^2 \text{ giving } k = \frac{y}{x^2}$$

Thus this family of curves satisfy the differential equation

$$\frac{dy}{dx} = 2\frac{y}{x^2}x = 2\frac{y}{x}.$$

Now using the fact that the product of the derivatives of two orthogonal curves meeting at a point must equal -1 , we get that the orthogonal trajectories satisfy the differential equation

$$\frac{dy}{dx} = \frac{-x}{2y}.$$

Separating the variables, we get

$$2ydy = -x dx$$

and

$$2 \int y dy = - \int x dx, \text{ or } y^2 = \frac{-x^2}{2} + C.$$

Hence our family of orthogonal trajectories is a family of curves of the form

$$y^2 + \frac{x^2}{2} = C,$$

a family of ellipses.

(a) $y = x^2 + C$

(b) $y^2 - x^2 = C$

(c) $y^2 + \frac{x^2}{2} = C$

(d) $y = Cx^2$

(e) $y^2 - \frac{x^2}{2} = C$

Name: _____

Instructor: _____

12.(7 pts.) The length of the curve $y = \frac{x^2}{8} - \ln(x)$, $1 \leq x \leq 2$, is given by:

$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx.$$

For

$$y = \frac{x^2}{8} - \ln(x), \quad 1 \leq x \leq 2,$$

we first compute

$$\frac{dy}{dx} = \frac{x}{4} - \frac{1}{x}.$$

Therefore,

$$L = \int_1^2 \sqrt{1 + \left(\frac{x}{4} - \frac{1}{x}\right)^2} dx.$$

Simplifying the expression inside the square root,

$$1 + \left(\frac{x}{4} - \frac{1}{x}\right)^2 = 1 + \frac{x^2}{16} - \frac{1}{2} + \frac{1}{x^2} = \frac{x^2}{16} + \frac{1}{2} + \frac{1}{x^2}.$$

Notice that

$$\frac{x^2}{16} + \frac{1}{2} + \frac{1}{x^2} = \left(\frac{x}{4} + \frac{1}{x}\right)^2.$$

Thus,

$$L = \int_1^2 \left| \frac{x}{4} + \frac{1}{x} \right| dx.$$

Since $x > 0$ on the interval $[1, 2]$,

$$\left| \frac{x}{4} + \frac{1}{x} \right| = \frac{x}{4} + \frac{1}{x}.$$

Hence,

$$L = \int_1^2 \left(\frac{x}{4} + \frac{1}{x} \right) dx.$$

Integrating,

$$L = \left[\frac{x^2}{8} + \ln(x) \right]_1^2.$$

Name: _____

Instructor: _____

Therefore,

$$L = \left(\frac{4}{8} + \ln 2 \right) - \left(\frac{1}{8} + \ln 1 \right) = \frac{3}{8} + \ln 2.$$

Thus the length of the curve is

$$\boxed{\frac{3}{8} + \ln 2}.$$

- (a) $\frac{3}{4}$
- (b) $\frac{\sqrt{2} - \ln(2)}{4}$
- (c) $\frac{3}{8} - \ln(2)$
- (d) $\frac{3}{8} + \ln(2)$
- (e) $\frac{1}{4}$

Name: _____

Instructor: _____

Partial Credit

You must show your work on the partial credit problems to receive credit!

13.(10 pts.) Find the integral

$$\int \frac{3x+1}{x^3+x^2} dx.$$

Solution: Use partial fraction decomposition

$$\frac{3x+1}{x^3+x^2} = \frac{3x+1}{x^2(x+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1} = \frac{Ax(x+1) + B(x+1) + Cx^2}{x^2(x+1)}.$$

Therefore

$$3x+1 = (A+C)x^2 + (A+B)x + B.$$

It follows that

$$\begin{aligned} A+C &= 0, & A+B &= 3, & B &= 1, \\ A &= 2, & B &= 1, & C &= -2, \end{aligned}$$

and

$$\int \frac{3x+1}{x^3+x^2} dx = \int \left(\frac{2}{x} + \frac{1}{x^2} - \frac{2}{x+1} \right) dx = 2 \ln |x| - \frac{1}{x} - 2 \ln |x+1| + C.$$

Name: _____

Instructor: _____

14.(10 pts.) (a) Circle the letter below alongside the trapezoidal approximation to

$$\ln 3 = \int_1^3 \frac{1}{x} dx \quad \text{using} \quad n = 8$$

A $\int_1^3 \frac{1}{x} dx \approx \frac{1}{8} \left[1 + 2\left(\frac{4}{5}\right) + 2\left(\frac{2}{3}\right) + 2\left(\frac{4}{7}\right) + 2\left(\frac{1}{2}\right) + 2\left(\frac{4}{9}\right) + 2\left(\frac{2}{5}\right) + 2\left(\frac{4}{11}\right) + \left(\frac{1}{3}\right) \right]$

B $\int_1^3 \frac{1}{x} dx \approx \frac{1}{12} \left[1 + 4\left(\frac{4}{5}\right) + 2\left(\frac{2}{3}\right) + 4\left(\frac{4}{7}\right) + 2\left(\frac{1}{2}\right) + 4\left(\frac{4}{9}\right) + 2\left(\frac{2}{5}\right) + 4\left(\frac{4}{11}\right) + \left(\frac{1}{3}\right) \right]$

C $\int_1^3 \frac{1}{x} dx \approx \frac{1}{8} \left[1 + \left(\frac{4}{5}\right) + \left(\frac{2}{3}\right) + \left(\frac{4}{7}\right) + \left(\frac{1}{2}\right) + \left(\frac{4}{9}\right) + \left(\frac{2}{5}\right) + \left(\frac{4}{11}\right) + \left(\frac{1}{3}\right) \right]$

(b) Recall that the error E_T in the trapezoidal rule for approximating $\int_a^b f(x) dx$ satisfies

$$\left| \int_a^b f(x) dx - T_n \right| = |E_T| \leq \frac{K(b-a)^3}{12n^2}$$

whenever $|f''(x)| \leq K$ for all $a \leq x \leq b$.

Use the above error bound to determine a value of n for which the trapezoidal approximation to $\ln 3 = \int_1^3 \frac{1}{x} dx$ has an error

$$|E_T| \leq \frac{1}{3} 10^{-4}.$$

$$f(x) = \frac{1}{x}, \quad f'(x) = \frac{-1}{x^2}, \quad f''(x) = \frac{2}{x^3}$$

Since $|f''(x)| = \frac{2}{x^3}$ is decreasing on the interval $1 \leq x \leq 2$, we have

$|f''(x)| \leq f''(1) = 2$ for $1 \leq x \leq 2$. Hence, we can use $K = 2$ in the error bound above.

For the trapezoidal approximation T_n , we have

$$|E_T| \leq \frac{K(b-a)^3}{12n^2} = \frac{2(3-1)^3}{12n^2} = \frac{16}{12n^2} = \frac{4}{3n^2}$$

If we find a value of n for which $\frac{1}{3} 10^{-4} \geq \frac{4}{3n^2}$, then we will have $|E_T| \leq \frac{1}{3} 10^{-4}$.

$$\frac{1}{3} 10^{-4} \geq \frac{4}{3n^2} \rightarrow n^2 \geq 4 \cdot 10^4 \rightarrow n \geq 2 \cdot 10^2 = 200$$

Name: _____

Instructor: _____

15. (6 pts.) Please circle “TRUE” if you think the statement is true, and circle “FALSE” if you think the statement is False.

(a)(1 pt. No Partial credit) The formula for the trapezoidal approximation with n approximating trapezoids is given by

$$\frac{\Delta x}{2} [2f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_n)].$$

TRUE

FALSE

(b))(1 pt. No Partial credit) $\int_1^\infty \frac{1}{x}$ diverges.

TRUE

FALSE

(c))(1 pt. No Partial credit) $\frac{dy}{dx} = \sqrt{x^2 + y^2}$ is a separable differential equation.

TRUE

FALSE

Name: _____

Instructor: _____

The following is the list of useful trigonometric formulas:

Note: $\sin^{-1} x$ and $\arcsin(x)$ are different names for the same function and $\tan^{-1} x$ and $\arctan(x)$ are different names for the same function.

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x$$

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$$

$$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$$

$$\sin 2x = 2 \sin x \cos x$$

$$\sin x \cos y = \frac{1}{2}(\sin(x - y) + \sin(x + y))$$

$$\sin x \sin y = \frac{1}{2}(\cos(x - y) - \cos(x + y))$$

$$\cos x \cos y = \frac{1}{2}(\cos(x - y) + \cos(x + y))$$

$$\int \sec \theta = \ln |\sec \theta + \tan \theta| + C$$

$$\int \csc \theta = \ln |\csc \theta - \cot \theta| + C$$

$$\csc \theta = \frac{1}{\sin \theta}, \quad \cot \theta = \frac{1}{\tan \theta}$$